

Dimension-six CP-violating operators of the third-family quarks and their effects at colliders

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ABSTRACT

We listed all possible dimension-six CP-violating $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariant operators involving the third-family quarks, which can be generated by new physics at a higher energy scale. The expressions of these operators after electroweak symmetry breaking and the induced effective couplings $Wt\bar{b}$, $Xb\bar{b}$ and $Xt\bar{t}$ ($X = Z, \gamma, g, H$) are also presented. We have evaluated sample contributions of these operators to CP-odd asymmetries of transverse polarization of top quark in single top production at the upgraded Tevatron, the similar effect in top-antitop pair production at the NLC, and the CP-odd observables of momentum correlations among the top quark decay products at the NLC. The energy and luminosity sensitivity in probing these CP-violating new physics has also been studied.

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1. Introduction

It is widely believed that the Standard Model (SM) is only an effective theory at electroweak scale and that some new physics should exist in higher energy regimes. Collider experiments have been searching for the new particles predicted by various models, but no direct signal has been observed. So, it is likely that the new particles are too heavy to be detectable at current colliders, and the only observable effects at energies not too far above the SM energy scale may be only in the form of new interactions. However, the new interactions will affect the couplings of third-family quarks, the Higgs and gauge bosons. In this spirit, the new physics effects can be expressed as non-standard terms in an effective Lagrangian involving the interactions of third-family quarks, the Higgs and gauge bosons. Before the electroweak symmetry breaking, we can write the effective Lagrangian as

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \frac{1}{\Lambda^2} \sum_i C_i O_i + \mathcal{O}(\frac{1}{\Lambda^4}) \quad (1)$$

where \mathcal{L}_0 is the SM Lagrangian, Λ is the new physics scale and O_i are $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariant dimension-six operators, and C_i are constants which represent the coupling strengths of O_i . The expansion in Eq.(1) was first discussed in Ref. [1]. Recently, many authors further classified such CP-conserving operators and analysed their phenomenological implications at current and future colliders[2-5].

As is well-known, for more than 30 years after the discovery of the CP-violating decays of the K_L^0 meson[6], the origin of this phenomenon remains a mystery. The SM gives a natural explanation for this phenomenon assuming the existence of a phase in the Kobayashi-Maskawa mixing matrix[7]. In models beyond the SM, additional CP-violating effects can appear rather naturally and such non-standard CP-violations are necessary in order to account for baryogenesis[8]. In Ref. [9], possible effects of non-SM CP-violating interactions have been studied in detail in the form of momentum space representation and involving only weak bosons. In this paper we will focus on CP-violation effects in the model-independent effective Lagrangian approach. So we assume that the new physics terms in Eq.(1) contain both CP-conserving and CP-violating operators.

It has been shown[10] that the KM mechanism of CP-violation predicts a negligibly small effect for the top quark in the SM, and thus the standard CP-violation effects in top production and decays will be unobservable in collider experiments. Therefore, top quark system will be sensitive to new source of CP-violation and may serve as a powerful probe to non-standard CP-violation in association with new physics effects. Non-standard CP-violation in the top quark system as predicted by various new physics models and the strategy for observing these effects have been studied by many authors[11-19]. Here we provide a model-independent study of all possible dimension-6 CP-violating operators which involve the third-family quarks and are invariant under the SM transformation. The effects of these operators can be studied at future linear and hadron colliders, and thus their strengths can be constrained. We will evaluate some of the effects of these CP-violating operators at the Tevatron and the NLC. Any nonzero value of these CP asymmetries will suggest the existence of new physics as well as new CP-violation effects.

This paper is organized as follows. In Sec.2 we list all possible dimension-six CP-violating $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariant operators. The expressions of these operators after electroweak gauge symmetry breaking are given in Appendix A. In Sec.3 we give the induced CP-violating effective couplings $Wt\bar{b}$, $Xb\bar{b}$ and $Xt\bar{t}$ ($X = Z, \gamma, g, H$). In Sec.4 we evaluate the contributions to some CP-odd quantities at the Tevatron and the NLC. And finally in Sec.5 we present the summary.

2. Dimension-six CP-violating gauge invariant operators

We assume that the new physics in the quark sector resides in the third quark family. Although new physics can give rise to four-quark operators involving only the third family, such operators are not experimentally relevant here. New physics may also occur in the gauge boson and Higgs sectors, they are not, however, our attention here. Therefore, the operators we are interested in are those containing third-family quarks coupling to gauge and Higgs bosons.

To restrict ourselves to the lowest order, we consider only tree diagrams and to the order of $1/\Lambda^2$. Therefore, only one vertex in a given diagram can contain anomalous couplings.

Under these conditions, operators which are allowed to be related by the field equations are not independent. As discussed in Ref.[5], to which we refer for the detail, the fermion and the Higgs boson equations of motion can be used but the equations of motion of the gauge bosons can not when writing down the operators in Eq.(1).

We assume all the operators O_i to be Hermitian. Because of our assumption that the available energies are below the unitarity cuts of new-physics particles, no imaginary part can be generated by the new physics effect. Therefore the coefficients C_i in Eq.(1) are real.

Now we list all possible dimension-six CP-odd $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariant operators involving third-family quarks but no four-fermion interactions. We follow the standard notation.

(1) Class 1 (contain t_R field)

$$O_{t1} = i(\Phi^\dagger \Phi - \frac{v^2}{2}) [\bar{q}_L t_R \tilde{\Phi} - \tilde{\Phi}^\dagger \bar{t}_R q_L] \quad (2)$$

$$O_{t2} = [\Phi^\dagger D_\mu \Phi + (D_\mu \Phi)^\dagger \Phi] \bar{t}_R \gamma^\mu t_R \quad (3)$$

$$O_{t3} = (\tilde{\Phi}^\dagger D_\mu \Phi) (\bar{t}_R \gamma^\mu b_R) + (D_\mu \Phi)^\dagger \tilde{\Phi} (\bar{b}_R \gamma^\mu t_R) \quad (4)$$

$$O_{Dt} = i [(\bar{q}_L D_\mu t_R) D^\mu \tilde{\Phi} - (D^\mu \tilde{\Phi})^\dagger (\overline{D_\mu t_R q_L})] \quad (5)$$

$$O_{tW\Phi} = i [(\bar{q}_L \sigma^{\mu\nu} \tau^I t_R) \tilde{\Phi} - \tilde{\Phi}^\dagger (\bar{t}_R \sigma^{\mu\nu} \tau^I q_L)] W_{\mu\nu}^I \quad (6)$$

$$O_{tB\Phi} = i [(\bar{q}_L \sigma^{\mu\nu} t_R) \tilde{\Phi} - \tilde{\Phi}^\dagger (\bar{t}_R \sigma^{\mu\nu} q_L)] B_{\mu\nu} \quad (7)$$

$$O_{tG\Phi} = i [(\bar{q}_L \sigma^{\mu\nu} T^A t_R) \tilde{\Phi} - \tilde{\Phi}^\dagger (\bar{t}_R \sigma^{\mu\nu} T^A q_L)] G_{\mu\nu}^A \quad (8)$$

$$O_{tG} = i [\bar{t}_R \gamma^\mu T^A D^\nu t_R - \overline{D^\nu t_R} \gamma^\mu T^A t_R] G_{\mu\nu}^A \quad (9)$$

$$O_{tB} = i [\bar{t}_R \gamma^\mu D^\nu t_R - \overline{D^\nu t_R} \gamma^\mu t_R] B_{\mu\nu} \quad (10)$$

(2) Class 2 (contain no t_R field)

$$O_{qG} = i [\bar{q}_L \gamma^\mu T^A D^\nu q_L - \overline{D^\nu q_L} \gamma^\mu T^A q_L] G_{\mu\nu}^A \quad (11)$$

$$O_{qW} = i [\bar{q}_L \gamma^\mu \tau^I D^\nu q_L - \overline{D^\nu q_L} \gamma^\mu \tau^I q_L] W_{\mu\nu}^I \quad (12)$$

$$O_{qB} = i [\bar{q}_L \gamma^\mu D^\nu q_L - \overline{D^\nu q_L} \gamma^\mu q_L] B_{\mu\nu} \quad (13)$$

$$O_{bG} = i [\bar{b}_R \gamma^\mu T^A D^\nu b_R - \overline{D^\nu b_R} \gamma^\mu T^A b_R] G_{\mu\nu}^A \quad (14)$$

$$O_{bB} = i [\bar{b}_R \gamma^\mu D^\nu b_R - \overline{D^\nu b_R} \gamma^\mu b_R] B_{\mu\nu} \quad (15)$$

$$O_{\Phi q}^{(1)} = [\Phi^\dagger D_\mu \Phi + (D_\mu \Phi)^\dagger \Phi] \bar{q}_L \gamma^\mu q_L \quad (16)$$

$$O_{\Phi q}^{(3)} = [\Phi^\dagger \tau^I D_\mu \Phi + (D_\mu \Phi)^\dagger \tau^I \Phi] \bar{q}_L \gamma^\mu \tau^I q_L \quad (17)$$

$$O_{\Phi b} = [\Phi^\dagger D_\mu \Phi + (D_\mu \Phi)^\dagger \Phi] \bar{b}_R \gamma^\mu b_R \quad (18)$$

$$O_{b1} = i(\Phi^\dagger \Phi - \frac{v^2}{2}) [\bar{q}_L b_R \Phi - \Phi^\dagger \bar{b}_R q_L] \quad (19)$$

$$O_{Db} = i[(\bar{q}_L D_\mu b_R) D^\mu \Phi - (D^\mu \Phi)^\dagger (\overline{D_\mu b_R q_L})] \quad (20)$$

$$O_{bW\Phi} = i[(\bar{q}_L \sigma^{\mu\nu} \tau^I b_R) \Phi - \Phi^\dagger (\bar{b}_R \sigma^{\mu\nu} \tau^I q_L)] W_{\mu\nu}^I \quad (21)$$

$$O_{bB\Phi} = i[(\bar{q}_L \sigma^{\mu\nu} b_R) \Phi - \Phi^\dagger (\bar{b}_R \sigma^{\mu\nu} q_L)] B_{\mu\nu} \quad (22)$$

$$O_{bG\Phi} = i[(\bar{q}_L \sigma^{\mu\nu} T^A b_R) \Phi - \Phi^\dagger (\bar{b}_R \sigma^{\mu\nu} T^A q_L)] G_{\mu\nu}^A \quad (23)$$

Note that in O_{t1} and O_{b1} we subtract the vacuum expectation value, $v^2/2$, from $\Phi^\dagger \Phi$, to avoid additional mass term for the third family quarks.

If we do not use the field equations of Higgs boson and the quarks, we would have the following additional operators

(3) Class 3

$$O_{\overline{Dt}} = i[(\overline{D_\mu q_L} t_R) D^\mu \tilde{\Phi} - (D^\mu \tilde{\Phi})^\dagger (\bar{t}_R D_\mu q_L)] \quad (24)$$

$$O_{\overline{Db}} = i[(\overline{D_\mu q_L} b_R) D^\mu \Phi - (D^\mu \Phi)^\dagger (\bar{b}_R D_\mu q_L)] \quad (25)$$

$$O_{t\tilde{G}} = [\bar{t}_R \gamma^\mu T^A D^\nu t_R + \overline{D^\nu t_R} \gamma^\mu T^A t_R] \tilde{G}_{\mu\nu}^A \quad (26)$$

$$O_{t\tilde{B}} = [\bar{t}_R \gamma^\mu D^\nu t_R + \overline{D^\nu t_R} \gamma^\mu t_R] \tilde{B}_{\mu\nu} \quad (27)$$

$$O_{q\tilde{G}} = [\bar{q}_L \gamma^\mu T^A D^\nu q_L + \overline{D^\nu q_L} \gamma^\mu T^A q_L] \tilde{G}_{\mu\nu}^A \quad (28)$$

$$O_{q\tilde{W}} = [\bar{q}_L \gamma^\mu \tau^I D^\nu q_L + \overline{D^\nu q_L} \gamma^\mu \tau^I q_L] \tilde{W}_{\mu\nu}^I \quad (29)$$

$$O_{q\tilde{B}} = [\bar{q}_L \gamma^\mu D^\nu q_L + \overline{D^\nu q_L} \gamma^\mu q_L] \tilde{B}_{\mu\nu} \quad (30)$$

$$O_{b\tilde{G}} = [\bar{b}_R \gamma^\mu T^A D^\nu b_R + \overline{D^\nu b_R} \gamma^\mu T^A b_R] \tilde{G}_{\mu\nu}^A \quad (31)$$

$$O_{b\tilde{B}} = [\bar{b}_R \gamma^\mu D^\nu b_R + \overline{D^\nu b_R} \gamma^\mu b_R] \tilde{B}_{\mu\nu}, \quad (32)$$

where $\tilde{X}_{\mu\nu} \equiv \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} X^{\lambda\rho}$ for $X = G, B, W$ and $\epsilon_{\mu\nu\lambda\rho}$ the anti-symmetric tensor. These Class 3 operators can be rewritten as

$$O_{\overline{Dt}} = -O_{Dt} - i[\bar{q}_L t_R D^2 \tilde{\Phi} - (D^2 \tilde{\Phi})^\dagger \bar{t}_R q_L], \quad (33)$$

$$O_{\overline{D}b} = -O_{Db} - i[\bar{q}_L b_R D^2 \Phi - (D^2 \tilde{\Phi})^\dagger \bar{b}_R q_L], \quad (34)$$

$$O_{x\tilde{B}} = -O_{xB} - (\bar{x}_R \sigma_{\mu\nu} \not{D} x_R + \overline{\not{D} x_R} \sigma_{\mu\nu} x_R) B^{\mu\nu}, \quad (x = t, b), \quad (35)$$

$$O_{x\tilde{G}} = -O_{xG} - (\bar{x}_R \sigma^{\mu\nu} T^a \not{D} x_R + \overline{\not{D} x_R} \sigma^{\mu\nu} T^a x_R) G_{\mu\nu}^a, \quad (x = t, b), \quad (36)$$

$$O_{q\tilde{X}} = O_{qX} + \bar{q}_L \sigma^{\mu\nu} X_{\mu\nu} \not{D} q_L + \overline{\not{D} q_L} \sigma^{\mu\nu} X_{\mu\nu} q_L, \quad (X = B, G, W). \quad (37)$$

They become dependent upon the use of the field equations of the Higgs boson and the quarks,

It should also be noted that those CP-violating operators which are obtained from Eqs.(6-8) and (21-23) by replacing the field tensors by their duals, $W_{\mu\nu}^a \rightarrow \widetilde{W}_{\mu\nu}^a$, etc., and changing the relative sign of the fermion operators are not independent due to the identity $\epsilon_{\mu\nu\lambda\rho} \sigma^{\lambda\rho} = 2i\sigma^{\mu\nu} \gamma_5$. For example, $[(\bar{q}_L \sigma^{\mu\nu} \tau^I t_R) \tilde{\Phi} + \tilde{\Phi}^\dagger (\bar{t}_R \sigma^{\mu\nu} \tau^I q_L)] \widetilde{W}_{\mu\nu}^I$ obtained from Eq.(6), is proportional to Eq.(6).

The expressions of these CP-violating operators Eqs.(2-23) after electroweak symmetry breaking are presented in Appendix A. Note that most of the operators clearly show the $U_{em}(1)$ gauge invariance. But some of them do not manifest the electroweak gauge invariance straight forwardly, for example, O_{Dt} in Eq.(A.4). We have checked that the operator gives indeed a $U_{em}(1)$ gauge invariant expression.

3. Effective Lagrangian for some couplings

We consider the contribution of CP-violating operators to top quark couplings $Wt\bar{b}$, $Zt\bar{t}$, $\gamma t\bar{t}$, $Ht\bar{t}$, $gt\bar{t}$ and the bottom quark coupling $Zb\bar{b}$, $\gamma b\bar{b}$. These couplings can be meaningfully investigated at LEP, Tevatron, NLC and LHC. The status of the contributions of the dimension-six CP-violating operators to these couplings are showed in Table 1.

Collecting all the relevant terms we get the CP-violating effective couplings as

$$\begin{aligned} \tilde{\mathcal{L}}_{Wtb} = & -i \frac{C_{\Phi q}^{(3)}}{\Lambda^2} \frac{g_2}{\sqrt{2}} v^2 W_\mu^+ (\bar{t} \gamma^\mu P_L b) - i \frac{C_{t3}}{\Lambda^2} \frac{v^2}{2} \frac{g_2}{\sqrt{2}} W_\mu^+ (\bar{t} \gamma^\mu P_R b) \\ & - i \frac{C_{Dt}}{\Lambda^2} \frac{v}{\sqrt{2}} \frac{g_2}{\sqrt{2}} W_\mu^+ (i \partial^\mu \bar{t}) P_L b - i \frac{C_{Db}}{\Lambda^2} \frac{v}{\sqrt{2}} \frac{g_2}{\sqrt{2}} W_\mu^+ \bar{t} P_R (i \partial^\mu b) \\ & - i \frac{C_{tW\Phi}}{\Lambda^2} \frac{v}{2} W_\mu^+ (\bar{t} \sigma^{\mu\nu} P_L b) + i \frac{C_{bW\Phi}}{\Lambda^2} \frac{v}{2} W_\mu^+ (\bar{t} \sigma^{\mu\nu} P_R b) \end{aligned}$$

$$+i\frac{C_{qW}}{\Lambda^2}\frac{1}{\sqrt{2}}W_{\mu\nu}^+[\bar{t}\gamma^\mu P_L(\partial^\nu b) - (\partial^\nu \bar{t})\gamma^\mu P_L b] + h.c. \quad (38)$$

$$\begin{aligned} \tilde{\mathcal{L}}_{Zb\bar{b}} = & i\left(\frac{C_{bW\Phi}}{\Lambda^2}\frac{c_W}{\sqrt{2}} + \frac{C_{bB\Phi}}{\Lambda^2}\frac{v}{\sqrt{2}}s_W\right)Z_{\mu\nu}(\bar{b}\sigma^{\mu\nu}\gamma_5 b) \\ & + i\left(\frac{C_{qW}}{\Lambda^2}\frac{c_W}{2} + \frac{C_{qB}}{\Lambda^2}s_W\right)Z_{\mu\nu}(\bar{b}\gamma^\mu P_L\partial^\nu b - \partial^\nu \bar{b}\gamma^\mu P_L b) \\ & + i\frac{C_{bB}}{\Lambda^2}s_W Z_{\mu\nu}(\bar{b}\gamma^\mu P_R\partial^\nu b - \partial^\nu \bar{b}\gamma^\mu P_R b) \\ & - i\frac{m_Z}{2}Z^\mu \left[i(\bar{b}\gamma_5\partial_\mu b - \partial_\mu \bar{b}\gamma_5 b)\frac{C_{Db}}{\Lambda^2} + i\partial_\mu(\bar{b}b)\frac{C_{Db}}{\Lambda^2} \right] \end{aligned} \quad (39)$$

$$\begin{aligned} \tilde{\mathcal{L}}_{\gamma b\bar{b}} = & i\left(\frac{C_{qB}}{\Lambda^2}c_W - \frac{C_{qW}}{\Lambda^2}\frac{s_W}{2}\right)A_{\mu\nu}(\bar{b}\gamma^\mu P_L\partial^\nu b - \partial^\nu \bar{b}\gamma^\mu P_L b) \\ & + i\frac{C_{bB}}{\Lambda^2}c_W A_{\mu\nu}(\bar{b}\gamma^\mu P_R\partial^\nu b - \partial^\nu \bar{b}\gamma^\mu P_R b) \\ & + i\left(\frac{C_{bB\Phi}}{\Lambda^2}c_W - \frac{C_{bW\Phi}}{\Lambda^2}\frac{s_W}{2}\right)\frac{v}{\sqrt{2}}A_{\mu\nu}(\bar{b}\sigma^{\mu\nu}\gamma_5 b) \end{aligned} \quad (40)$$

$$\begin{aligned} \tilde{\mathcal{L}}_{Zt\bar{t}} = & i\frac{C_{Dt}}{\Lambda^2}\frac{1}{\sqrt{2}}\frac{m_Z}{2}Z^\mu[i\partial_\mu(\bar{t}t)] + i\frac{C_{Dt}}{\Lambda^2}\frac{1}{\sqrt{2}}\frac{m_Z}{2}Z^\mu(i\bar{t}\gamma_5\partial_\mu t - i\partial_\mu \bar{t}\gamma_5 t) \\ & + i\left(\frac{C_{tB\Phi}}{\Lambda^2}s_W - \frac{C_{tW\Phi}}{\Lambda^2}\frac{c_W}{2}\right)\frac{v}{\sqrt{2}}Z_{\mu\nu}(\bar{t}\sigma^{\mu\nu}\gamma_5 t) \\ & + i\frac{C_{tB}}{\Lambda^2}s_W Z_{\mu\nu}(\bar{t}\gamma^\mu P_R\partial^\nu t - \partial^\nu \bar{t}\gamma^\mu P_R t) \\ & + i\left(\frac{C_{qB}}{\Lambda^2}s_W - \frac{C_{qW}}{\Lambda^2}\frac{c_W}{2}\right)Z_{\mu\nu}(\bar{t}\gamma^\mu P_L\partial^\nu t - \partial^\nu \bar{t}\gamma^\mu P_L t) \end{aligned} \quad (41)$$

$$\begin{aligned} \tilde{\mathcal{L}}_{\gamma t\bar{t}} = & i\left(\frac{C_{tW\Phi}}{\Lambda^2}\frac{s_W}{2} + \frac{C_{tB\Phi}}{\Lambda^2}c_W\right)\frac{v}{\sqrt{2}}A_{\mu\nu}(\bar{t}\sigma^{\mu\nu}\gamma_5 t) \\ & + i\frac{C_{tB}}{\Lambda^2}c_W A_{\mu\nu}(\bar{t}\gamma^\mu P_R\partial^\nu t - \partial^\nu \bar{t}\gamma^\mu P_R t) \\ & + i\left(\frac{C_{qB}}{\Lambda^2}c_W + \frac{C_{qW}}{\Lambda^2}\frac{s_W}{2}\right)A_{\mu\nu}(\bar{t}\gamma^\mu P_L\partial^\nu t - \partial^\nu \bar{t}\gamma^\mu P_L t) \end{aligned} \quad (42)$$

$$\begin{aligned} \tilde{\mathcal{L}}_{Ht\bar{t}} = & i\frac{C_{t1}}{\Lambda^2}\frac{v^2}{\sqrt{2}}H(\bar{t}\gamma_5 t) + i\frac{C_{Dt}}{\Lambda^2}\frac{1}{2\sqrt{2}}\partial^\mu H[\partial_\mu(\bar{t}\gamma_5 t) + \bar{t}\partial_\mu t - (\partial_\mu \bar{t})t] \\ & - i\frac{C_{t2}}{\Lambda^2}v(i\partial^\mu H)(\bar{t}\gamma_\mu P_R t) - i\left(\frac{C_{\Phi q}^{(1)}}{\Lambda^2} - \frac{C_{\Phi q}^{(3)}}{\Lambda^2}\right)v(i\partial^\mu H)(\bar{t}\gamma_\mu P_L t) \end{aligned} \quad (43)$$

$$\begin{aligned} \tilde{\mathcal{L}}_{gt\bar{t}} = & i\frac{C_{tG}}{\Lambda^2}[\bar{t}\gamma^\mu P_R T^A\partial^\nu t - \partial^\nu \bar{t}\gamma^\mu P_R T^A t]G_{\mu\nu}^A \\ & + i\frac{C_{qG}}{\Lambda^2}[\bar{t}\gamma^\mu P_L T^A\partial^\nu t - \partial^\nu \bar{t}\gamma^\mu P_L T^A t]G_{\mu\nu}^A \\ & + i\frac{C_{tG\Phi}}{\Lambda^2}\frac{v}{\sqrt{2}}(\bar{t}\sigma^{\mu\nu}\gamma_5 T^A t)G_{\mu\nu}^A \end{aligned} \quad (44)$$

$$\begin{aligned}
\tilde{\mathcal{L}}_{Hb\bar{b}} = & -i\frac{1}{\Lambda^2}(C_{\Phi q}^{(1)} + C_{\Phi q}^{(3)})v(i\partial_\mu H)\bar{b}\gamma^\mu P_L b - i\frac{C_{\Phi b}}{\Lambda^2}v(i\partial_\mu H)\bar{b}\gamma^\mu P_R b \\
& + i\frac{C_{b1}}{\Lambda^2}\frac{v^2}{\sqrt{2}}H(\bar{b}\gamma_5 b) + i\frac{C_{Db}}{\Lambda^2}\frac{1}{2\sqrt{2}}\partial^\mu H\left[\bar{b}\partial_\mu b - (\partial_\mu \bar{b})b + \partial_\mu(\bar{b}\gamma_5 b)\right], \quad (45)
\end{aligned}$$

where $s_W \equiv \sin \theta_W$, $c_W \equiv \cos \theta_W$ and $P_{L,R} \equiv (1 \mp \gamma_5)/2$.

4. The contributions to CP-odd quantities of top quark at colliders

Various experiments have been suggested to measure CP-violating couplings of the top quark. They include CP-odd quantities such as the polarization asymmetries[12-14] and CP-odd momentum correlations among the decay products[15,16].

In this section we will evaluate the contributions of some of the CP-violating new physics operators to these CP asymmetries. By taking individual operator as an example, we present numerical results to show at what level of C_i/Λ^2 the CP-violating effect may be visible. We will only consider the CP-odd operators listed in Sec.3 and do not include their corresponding CP-even operators whose phenomenologies are different and have been systematically analysed in Refs.[3-5]. Further more, we restrict ourselves to the electroweak vertices, i.e., Wtb , $Zt\bar{t}$ and $\gamma t\bar{t}$.

4.1 Transverse polarization ² asymmetry of top quark in single top production at the Tevatron

The reaction $p\bar{p} \rightarrow t\bar{b}X$ at the Tevatron can be used to investigate several different types of CP asymmetries[15]. The complicate coordinate representation of the effective Lagrangian Eqs.(38-45) can be simplified in the momentum space when t and b are on-shell. The CP-violating contribution to the Wtb vertex Eqs.(38) can be written in the momentum space as

$$\begin{aligned}
\tilde{\mathcal{L}}_{Wtb} = & i\frac{g_2}{\sqrt{2}}W_\mu^+\bar{t}\left[F_L\gamma^\mu P_L + F_R\gamma^\mu P_R - i\frac{G_L}{m_t}\sigma^{\mu\nu}k_\nu P_L - i\frac{G_R}{m_t}\sigma^{\mu\nu}k_\nu P_R\right]b \\
& - i\frac{g_2}{\sqrt{2}}W_\mu^-\bar{b}\left[F_L\gamma^\mu P_L + F_R\gamma^\mu P_R - i\frac{G_L}{m_t}\sigma^{\mu\nu}k_\nu P_R - i\frac{G_R}{m_t}\sigma^{\mu\nu}k_\nu P_L\right]t, \quad (46)
\end{aligned}$$

²In this paper the transverse polarization direction is the one which is perpendicular to the scattering plane.

where $P_{L,R} \equiv (1 \mp \gamma_5)/2$, $k = p_t + p_{\bar{b}}$, and

$$F_L = \frac{v^2}{\Lambda^2} [-C_{\Phi q}^{(3)} + \frac{C_{Dt}}{2\sqrt{2}} \frac{m_t}{v}], \quad (47)$$

$$G_L = \frac{v^2}{\Lambda^2} [\frac{C_{Dt}}{2\sqrt{2}} \frac{m_t}{v} + C_{tW\Phi} \frac{\sqrt{2}}{g_2} \frac{m_t}{v} - C_{qW} \frac{1}{g_2} \frac{m_t^2}{v^2}], \quad (48)$$

$$F_R = -\frac{v^2}{2\Lambda^2} [C_{t3} + \frac{C_{Db}}{\sqrt{2}} \frac{m_t}{v}], \quad (49)$$

$$G_R = -\frac{v^2}{\Lambda^2} [\frac{C_{Db}}{2\sqrt{2}} \frac{m_t}{v} + C_{tW\Phi} \frac{\sqrt{2}}{g_2} \frac{m_t}{v}], \quad (50)$$

We have neglected the scalar and pseudoscalar couplings, k_μ and $k_\mu \gamma_5$, which, in the process $u\bar{d} \rightarrow W \rightarrow t\bar{b}$, give contributions proportional to the initial parton mass. It should be pointed out that in contrast to Ref.[15], where the form factors F_L , etc., can be complex, form factors in Eq.(46) are all real because $C_{\Phi q}^{(3)}$, etc., are real as noted in Sec.2 above.

The spin of the top quark allows three types of CP-violating polarization asymmetries [15] in the single top quark production via

$$u + \bar{d} \rightarrow t + \bar{b}, \quad \bar{u} + d \rightarrow \bar{t} + b. \quad (51)$$

Introducing the coordinate system in the top quark (or top antiquark) rest frame with the unit vectors $\vec{e}_z \propto -\vec{P}_{\bar{b}}$, $\vec{e}_y \propto \vec{P}_u \times \vec{P}_{\bar{b}}$ and $\vec{e}_x = \vec{e}_y \times \vec{e}_z$, the transverse polarization asymmetry is defined as

$$A(\hat{y}) = \frac{1}{2} [\Pi(\hat{y}) - \bar{\Pi}(\hat{y})], \quad (52)$$

where $\Pi(\hat{y})$ and $\bar{\Pi}(\hat{y})$ are, respectively, the polarizations of the top quark and top antiquark in the direction \hat{y} , arising from the interference of the SM and the CP-violating vertices. Only the terms proportional to P_L contribute. The polarizations are given by

$$\Pi(\hat{y}) = \frac{N_t(+\hat{y}) - N_t(-\hat{y})}{N_t(+\hat{y}) + N_t(-\hat{y})}, \quad (53)$$

$$\bar{\Pi}(\hat{y}) = \frac{N_{\bar{t}}(+\hat{y}) - N_{\bar{t}}(-\hat{y})}{N_{\bar{t}}(+\hat{y}) + N_{\bar{t}}(-\hat{y})}, \quad (54)$$

where $N_t(\pm\hat{y})$ [$N_{\bar{t}}(\pm\hat{y})$] is the number of $t(\bar{t})$ quarks polarized in the direction $\pm\hat{y}$.

The asymmetry $A(\hat{y})$ is proportional to the real part of the form factor G_L , which is given by[15]

$$A(\hat{y}) = \frac{3\pi}{4} \frac{(1-x)}{(2+x)\sqrt{x}} \text{Re } G_L, \quad (55)$$

where $x = m_t^2/\hat{s}$. This parton level asymmetry can be converted to the hadron level asymmetry by folding in the structure functions. In the absence of an imaginary part F_L makes no contribution to polarization asymmetries.

Using the CTEQ3L parton distribution functions[20] with $\mu = \sqrt{\hat{s}}$ and assuming $m_t = 175$ GeV, we obtain the asymmetry as

$$A(\hat{y}) = \begin{cases} -0.41 \frac{C_{qW} - 2C_{tW\Phi} - g_2 C_{Dt/2}}{(\Lambda/1 \text{ TeV})^2} & \text{at } \sqrt{s} = 2 \text{ TeV} \\ -0.84 \frac{C_{qW} - 2C_{tW\Phi} - g_2 C_{Dt/2}}{(\Lambda/1 \text{ TeV})^2} & \text{at } \sqrt{s} = 4 \text{ TeV} \end{cases} \quad (56)$$

As analysed in Ref.[15], such an asymmetry of a few percent might be within the reach of experiment at the upgraded Tevatron with $\sqrt{s} = 2$ TeV and an integrated luminosity 3-10 fb⁻¹. As the results in Eq.(56) show, the CP asymmetry caused by new physics will be more significant at higher energies, say $\sqrt{s} = 4$ TeV. Hence, if the collider can be further upgraded to 4 TeV and/or with increased luminosity[21], it can serve as a more powerful tool for probing CP-violating new physics. It should be noted that the signal for this process is unobservable at the LHC because of the large background from $t\bar{t}$ production and single top production via W -gluon fusion[22].

Let's take O_{qW} as an example. If we assume an observable level of ten percent, we see from Eq.(56) that the upgraded Tevatron will probe $\frac{C_{qW}}{(\Lambda/1 \text{ TeV})^2}$ to 1/4 and 1/8 for $\sqrt{s} = 2$ TeV and $\sqrt{s} = 4$ TeV, respectively. This means that with a new physics scale at the order of 1 TeV, the further upgraded Tevatron can probe the coupling strength down to the level of 0.1.

4.2 Transverse polarization asymmetry of top quark pair production at the NLC

From the polarizations of the top quark and top antiquark in $e^+e^- \rightarrow t\bar{t}$, one can construct CP-odd quantities which can be measured through the energy asymmetry of the charged leptons

in the t and \bar{t} decays as well as the up-down asymmetry of these leptons with respect to the $t\bar{t}$ production plane[12,13].

Including both the SM couplings and new physics effects, we can write the $Vt\bar{t}$ ($V = Z, \gamma$) vertices as

$$\Gamma_{Vt\bar{t}}^\mu = i\frac{g}{2} \left[\gamma_\mu A_V - \gamma_\mu \gamma_5 B_V + \frac{p_t^\mu - p_{\bar{t}}^\mu}{2} (C_V - iD_V \gamma_5) \right], \quad (57)$$

where p_t and $p_{\bar{t}}$ are the momenta of the top quark and top antiquark. We neglect the scalar and pseudoscalar couplings, k_μ and $k_\mu \gamma_5$ with $k = p_t + p_{\bar{t}}$, since these terms give contributions proportional to the electron mass. We note that some of these neglected terms are needed to maintain the electromagnetic gauge invariance for the axial vector couplings in Eq.(57). The form factors can be written as

$$X_V = X_V^{\text{SM}} + \delta X_V, \quad (X = A, B, C, D \text{ and } V = Z, \gamma), \quad (58)$$

where X_V^{SM} and δX_V represent the SM and the new physics contributions, respectively. In the SM, only $A_{\gamma,Z}$ and B_Z exist at tree level. Beyond the tree level, all of them except the CP-violating form factor D get contributions from loop diagrams. The SM loop contribution to D is completely negligible[10]. Since we are interested in CP-violation effect, we neglect the SM loop contributions to all form factors. Thus we have

$$A_\gamma^{\text{SM}} = \frac{4}{3}s_W, \quad A_Z^{\text{SM}} = \frac{1}{2c_W}(1 - \frac{8}{3}s_W^2), \quad (59)$$

$$B_\gamma^{\text{SM}} = 0, \quad B_Z^{\text{SM}} = \frac{1}{2c_W}, \quad (60)$$

$$C_\gamma^{\text{SM}} = D_\gamma^{\text{SM}} = C_Z^{\text{SM}} = D_Z^{\text{SM}} = 0. \quad (61)$$

For new physics effects, only the form factor D receives CP-violating contributions. Then we obtain

$$\delta A_{\gamma,Z} = \delta B_{\gamma,Z} = \delta C_{\gamma,Z} = 0, \quad (62)$$

$$\delta D_\gamma = -\frac{v}{\Lambda^2} \frac{4}{g} [(C_{qB} - C_{tB}) \frac{c_W m_t}{v} + C_{qW} \frac{s_W m_t}{2v} - C_{tW\Phi} \frac{s_W}{\sqrt{2}} - C_{tB\Phi} \sqrt{2} c_W], \quad (63)$$

$$\delta D_Z = \frac{v}{\Lambda^2} \frac{4}{g} [(C_{qB} - C_{tB}) \frac{s_W m_t}{v} - C_{qW} \frac{c_W m_t}{2v} + C_{tW\Phi} \frac{c_W}{\sqrt{2}} - C_{tB\Phi} \sqrt{2} s_W + C_{Dt} \frac{m_Z}{2\sqrt{2}v}] \quad (64)$$

The nonvanishing real parts of D can give rise to the following asymmetry[14]

$$A_T = P_\perp \sin \alpha - \bar{P}_\perp \sin \bar{\alpha}, \quad (65)$$

where $P_\perp \sin \alpha$ ($\bar{P}_\perp \sin \bar{\alpha}$) is the degree of transverse polarization of the t (\bar{t}) quark perpendicular to the scattering plane of $e^+e^- \rightarrow t\bar{t}$. The scattering plane is defined to be the X - Z plane where the $+Z$ direction is the direction of electron and the top-quark momentum has a positive x -component. The angle α depends on the top quark polarization direction and its definition can be found in Appendix C of the first article of Ref. 14. $P_\perp \sin \alpha$ and $\bar{P}_\perp \sin \bar{\alpha}$ are given by

$$P_\perp \sin \alpha = \frac{T_\perp}{G}, \quad \bar{P}_\perp \sin \bar{\alpha} = \frac{\bar{T}_\perp}{G}, \quad (66)$$

where

$$G = |(+ - ++)|^2 + |(+ - + -)|^2 + |(+ - - +)|^2 + |(+ - - -)|^2 \\ + |(- + ++)|^2 + |(- + + -)|^2 + |(- + - +)|^2 + |(- + - -)|^2, \quad (67)$$

$$T_\perp = 2 \operatorname{Im} [(+ - ++)^*(+ - - +) + (+ - + -)^*(+ - - -) \\ + (- + ++)^*(- + - +) + (- + + -)^*(- + - -)], \quad (68)$$

$$\bar{T}_\perp = 2 \operatorname{Im} [(+ - ++)^*(+ - + -) + (+ - - +)^*(+ - - -) \\ + (- + ++)^*(- + + -) + (- + - +)^*(- + - -)]. \quad (69)$$

Here the helicity amplitudes $(h_{e^-}, h_{e^+}, h_t, h_{\bar{t}})$, where $h_{e^-} = -, +$, etc., indicate respectively a left- and right-handed electron, etc., are given by

$$(h_{e^-}, h_{e^+}, h_t, h_{\bar{t}}) = 2g^2 E \left[\frac{(h_{e^-}, h_{e^+}, h_t, h_{\bar{t}})_Z}{s - M_Z^2} + \frac{(h_{e^-}, h_{e^+}, h_t, h_{\bar{t}})_\gamma}{s} \right]. \quad (70)$$

The nonvanishing $(h_{e^-}, h_{e^+}, h_t, h_{\bar{t}})_V$ ($V = \gamma, Z$) can be found in Ref. 14 and are listed below:

$$(- + - -)_V = e_L^V \sin \theta_t (m_t A_V - K^2 C_V + i E K D_V), \quad (71)$$

$$(- + - +)_V = -e_L^V (1 + \cos \theta_t) (E A_V + K B_V), \quad (72)$$

$$(- + + -)_V = e_L^V (1 - \cos \theta_t) (E A_V - K B_V), \quad (73)$$

$$(- + + +)_V = e_L^V \sin \theta_t (-m_t A_V + K^2 C_V + i E K D_V), \quad (74)$$

$$(+---)_V = e_R^V \sin \theta_t (m_t A_V - K^2 C_V + i E K D_V), \quad (75)$$

$$(+--+)_V = e_R^V (1 - \cos \theta_t) (E A_V + K B_V), \quad (76)$$

$$(+ - + -)_V = -e_R^V (1 + \cos \theta_t) (E A_V - K B_V), \quad (77)$$

$$(+ - ++)_V = e_R^V \sin \theta_t (-m_t A_V + K^2 C_V + i E K D_V), \quad (78)$$

where θ_t is the angle between the top quark and the electron, $E = \sqrt{s}/2$, $K = \sqrt{E^2 - m_t^2}$ and $e_{L,R}^V$ are the form factors in $V e^- e^+$ vertex $i g \gamma^\mu (e_L^V P_L + e_R^V P_R)$, which are given by

$$e_L^Z = \frac{1}{c_W} \left(-\frac{1}{2} + s_W^2 \right), \quad e_R^Z = \frac{1}{c_W} s_W^2, \quad (79)$$

$$e_L^\gamma = e_R^\gamma = -s_W. \quad (80)$$

As in the preceding subsection, we take the operator O_{qW} as an example to show the numerical results. Assuming the coupling strength $C_{qW} = 0.1$, the asymmetry A_T as a function of θ_t in the top pair production at the NLC is plotted in Fig.1 and Fig.2 for $\sqrt{s} = 500$ GeV and $\sqrt{s} = 1$ TeV, respectively. Figure 1 shows that if the scale of new physics which generates the operator O_{qW} is below 1.5 TeV, the A_T induced can exceed one percent. Comparing Fig.1 with Fig.2, we find that the asymmetry A_T for $\sqrt{s} = 1$ TeV is larger than that for $\sqrt{s} = 500$ GeV. To see more clearly, we compare the values corresponding to $\theta_t = 120^\circ$

$\Lambda(\text{TeV})$	0.5	1	1.5	2
$A_T(\%)$ ($\sqrt{s} = 0.5$ TeV)	-9.97	-2.50	-1.11	-0.62
$A_T(\%)$ ($\sqrt{s} = 1$ TeV)	-36.87	-9.34	-4.15	-2.34

Here we see that the A_T for $\sqrt{s} = 1$ TeV is four times larger than that for $\sqrt{s} = 500$ GeV. But since the total event rate at a 1 TeV machine is about four times smaller than a 500 GeV machine, the net effect is that a 1 TeV machine cannot provide a better measurement unless it has a higher luminosity.

4.3 Momentum correlations among the decay products of top quark at the NLC

In the process $e^+e^- \rightarrow \gamma^*, Z^* \rightarrow t\bar{t}$ with $t \rightarrow W^+b$ and $\bar{t} \rightarrow W^-\bar{b}$, some CP-odd momentum correlations among the decay products can be constructed [15,16]. One of them, which is *CPT*-even and sensitive to the real part of the dipole moment factor D in Eq.(57), is

$$O_1 = (\vec{p}_b \times \vec{p}_{\bar{b}}) \cdot \hat{e}_z, \quad (81)$$

where \hat{e}_z is the unit vector along the incoming positron beam direction. However, this observable is not sensitive to possible CP violation of the $t\bar{t}W$ vertex in the top quark decay [15,16]. Thus we consider only the CP-violating new physics effects in the vertices $Vt\bar{t}$ ($V = \gamma, Z$). In terms of the expression Eq.(57), one gets the average value [17]

$$\begin{aligned} \langle O_1 \rangle = & -\frac{g}{48}sm_t(1-x)\epsilon^2\beta\Sigma^{-1} \left\{ \frac{1}{s^2}C^{\gamma\gamma}(v_e^\gamma)^2v_t^\gamma\text{Re}D_\gamma \right. \\ & + \frac{1}{s(s-m_Z^2)}C^{Z\gamma}v_e^\gamma v_e^Z(v_t^Z - \frac{\beta}{3}a_t^Z)\text{Re}D_\gamma \\ & + \frac{1}{s(s-m_Z^2)}C^{Z\gamma}v_e^\gamma v_e^Zv_t^\gamma\text{Re}D_Z \\ & \left. + \frac{1}{(s-m_Z^2)^2}C^{ZZ}[(v_e^Z)^2 + (a_e^Z)^2](v_t^Z - \frac{\beta}{3}a_t^Z)\text{Re}D_Z \right\}, \end{aligned} \quad (82)$$

where

$$\begin{aligned} x &= \frac{4m_t^2}{s}, \quad \epsilon = 1 - \frac{m_W^2}{m_t^2}, \\ \beta &= \frac{m_t^2 - 2m_W^2}{m_t^2 + 2m_W^2}, \\ C^{\gamma\gamma} &= -p, \quad C^{Z\gamma} = \frac{a_e^Z}{v_e^Z} - p, \\ C^{ZZ} &= \frac{2a_e^Z v_e^Z}{(v_e^Z)^2 + (a_e^Z)^2} - p, \\ v_e^V &= \frac{1}{2s_W}(e_L^V + e_R^V), \quad v_t^V = \frac{A_V}{2s_W}, \\ a_e^V &= \frac{B_V}{2s_W}, \quad a_t^V = \frac{1}{2s_W}(e_L^V - e_R^V), \end{aligned} \quad (83)$$

and

$$\begin{aligned} \Sigma = & \frac{1}{s^2}(1 + \frac{x}{2})(v_e^\gamma)^2(v_t^\gamma)^2 + \frac{2}{s(s-m_Z^2)}(1 + \frac{x}{2})v_e^\gamma v_t^\gamma(v_e^Z - pa_e^Z)v_t^Z \\ & + \frac{1}{(s-m_Z^2)^2}[(v_e^Z)^2 + (a_e^Z)^2 - 2pv_e^Z a_e^Z] \left[(1 + \frac{x}{2})(v_t^Z)^2 + (1-x)(a_t^Z)^2 \right]. \end{aligned} \quad (84)$$

In the above equations, s is the center-of-mass energy squared and p is the degree of longitudinal polarization of the initial electron with $p = \pm 1$ corresponding to the right- and left-handed helicities, respectively. Note that in our analyses we neglect both the radiative corrections to the couplings Ve^+e^- ($V = \gamma, Z$) and the electron mass, thus only the left-right and right-left combinations³ of electron and positron helicities couple to the γ and Z .

Again we take the operator O_{qW} as an example to show some results. The values of $\langle O_1 \rangle$ for different polarizations of the electron beam with new physics scale of 1 TeV and coupling strength of unity are found to be

	$e^+e_L^-$	$e^+e_R^-$	e^+e^-
$\langle O_1 \rangle [(\text{GeV})^2]$ ($\sqrt{s} = 0.5 \text{ TeV}$)	-36.7	-1.0	-25.5
$\langle O_1 \rangle [(\text{GeV})^2]$ ($\sqrt{s} = 1 \text{ TeV}$)	-272.3	-1.7	-183.4

Here we find that the left-polarized electron beam yields the most significant results for $\langle O_1 \rangle$ and in this case the result in a 1 TeV accelerator is eight times larger than a 500 GeV accelerator. In the following analyses we will only consider the left-polarized electron beam.

Now we compare the value of $\langle O_1 \rangle$ with the expected variance $\langle O_1^2 \rangle$ to see what luminosity is needed for the observation to be statistically significant. To observe a deviation from the SM expectation with better than one standard deviation (at the 68% confidence level), we need

$$|\langle O_1 \rangle| \geq \sqrt{\frac{\langle O_1^2 \rangle}{\mathcal{L}\sigma\kappa}}, \quad (85)$$

where \mathcal{L} is the integrated luminosity, κ is the overall b - and W -tagging efficiency. The variance $\langle O_1^2 \rangle$ and the production cross section σ at lowest order are given by[17]

$$\begin{aligned} \sigma &= 4\pi\alpha^2 s \sqrt{1-x} \Sigma, \\ \langle O_1^2 \rangle &= \frac{sm_t^2 \epsilon^4}{2880} \Sigma^{-1} \left\{ \frac{1}{s^2} (v_e^\gamma)^2 (v_t^\gamma)^2 [24 + 2x - 11x^2 + 4\beta^2(1-x)^2] \right. \\ &\quad \left. + \frac{2}{s(s-m_Z^2)} v_e^\gamma v_e^Z \left[v_t^\gamma v_t^Z (24 + 2x - 11x^2 + 4\beta^2(1-x)^2) - 2v_t^\gamma a_t^Z (1-x)(6-x)\beta \right] \right\} \end{aligned} \quad (86)$$

³Hard collinear emission of a photon from the electron and positron beams can flip helicities. This gives rise to non-zero CP-odd correlations even in the absence of CP-violating interactions and this background should be subtracted. However, as analysed in Ref.[17], there will be no such background at tree level for $\langle O_1 \rangle$.

$$\begin{aligned}
& + \frac{1}{(s - m_Z^2)^2} [(v_e^Z)^2 + (a_e^Z)^2] \left[(v_t^Z)^2 (24 + 2x - 11x^2 + 4\beta^2(1-x)^2) \right. \\
& \left. + (a_t^Z)^2 (24 - 14x - 4\beta^2(1-x)) (1-x) - 4v_t^Z a_t^Z (1-x)(6-x)\beta \right] \}. \quad (87)
\end{aligned}$$

For a negative helicity electron beam considered in our analyses, the production rate is

$$\sigma(e^+ e_L^- \rightarrow t\bar{t}) = \begin{cases} 775 \text{ fb} & \text{for } \sqrt{s} = 500 \text{ GeV}, \\ 232 \text{ fb} & \text{for } \sqrt{s} = 1 \text{ TeV}. \end{cases} \quad (88)$$

Assuming the coupling strength of the order of unity and an overall b - and W -tagging efficiency of 50%, then the luminosity required to observe the CP-violating effects of O_{qW} at 68% confidence level is found to be

$$\mathcal{L} = \begin{cases} 25 \frac{(\Lambda/1 \text{ TeV})^4}{C_{qW}^2} \text{ fb}^{-1} & \text{at } \sqrt{s} = 0.5 \text{ TeV} \\ 8 \frac{(\Lambda/1 \text{ TeV})^4}{C_{qW}^2} \text{ fb}^{-1} & \text{at } \sqrt{s} = 1 \text{ TeV} \end{cases} \quad (89)$$

So, if the new physics scale is 1 TeV, we need a luminosity of 100 fb^{-1} (30 fb^{-1}) to probe the coupling strength C_{qW} down to 0.5 with a confidence level of 68% at $\sqrt{s} = 500 \text{ GeV}$ (1 TeV). If a conservative overall b - and W -tagging efficiency of 10% is assumed, the required luminosity will be increased by a factor of 5. If a confidence level of 99.7% is assumed, the required luminosity will be increased by a factor of 9.

From the above results we find that for the same luminosity a 1 TeV collider can do a better measurement than a 500 GeV collider. This is due to the fact that the size of $\langle O_1 \rangle$ at $\sqrt{s} = 1 \text{ TeV}$ is eight times larger than at $\sqrt{s} = 500 \text{ GeV}$, while the production rate at $\sqrt{s} = 1 \text{ TeV}$ is only about four times smaller than at $\sqrt{s} = 500 \text{ GeV}$. Thus the net effect is that a 1 TeV accelerator can do a better measurement than a 500 GeV accelerator.

5. Summary

In this paper we listed all possible dimension-six CP-violating $SU_c(3) \times SU_L(2) \times U_Y(1)$ invariant operators involving the third-family quarks, which may be generated by new physics at a higher scale. The expressions of these operators after the electroweak symmetry breaking and the induced effective couplings for $Wt\bar{b}$, $Vb\bar{b}$ and $Vt\bar{t}$ ($V = Z, \gamma, g, H$) were presented.

The contributions of some of these operators to the CP-odd asymmetries of the transverse polarization of top quark and top antiquark in single top production at the Tevatron and top

pair production at the NLC are evaluated. The numerical results showed that if the new physics scale is around 1 TeV, then both colliders can be used to probe the coupling strength to 0.1 provided that the asymmetry of the transverse polarization can be measured at a level of a few percent.

We also calculated the effects on a CP-odd observable, which involves momentum correlations among the decay products of the top quark, at the NLC and studied the dependence on the energy and luminosity of the NLC. We found that with a luminosity of 100 fb^{-1} , a 500 GeV accelerator can probe the coupling strength to 0.5, assuming that the new physics scale is of the order of 1 TeV. Achieving the same measurement, we need a luminosity of 30 fb^{-1} at a 1 TeV accelerator.

Acknowledgement

J.M.Y. thanks C.-P.Yuan for discussions. This work was supported in part by the U.S. Department of Energy, Division of High Energy Physics, under Grant No. DE-FG02-94ER40817.

Appendix A CP-violating operators after electroweak symmetry breaking

(1) Class 1

$$O_{t1} = \frac{1}{2\sqrt{2}}H(H+2v)(H+v)(\bar{t}i\gamma_5t) \quad (\text{A.1})$$

$$O_{t2} = (H+v)\partial^\mu H(\bar{t}_R\gamma_\mu t_R) \quad (\text{A.2})$$

$$O_{t3} = i\frac{1}{2\sqrt{2}}g_2(H+v)^2 \left[-W_\mu^+(\bar{t}_R\gamma^\mu b_R) + W_\mu^-(\bar{b}_R\gamma^\mu t_R) \right] \quad (\text{A.3})$$

$$\begin{aligned} O_{Dt} = & i\frac{1}{2\sqrt{2}}\partial^\mu H \left[\bar{t}\partial_\mu t - (\partial_\mu \bar{t})t + \partial_\mu(\bar{t}\gamma_5 t) - i\frac{4}{3}g_1 B_\mu \bar{t}t \right] \\ & - \frac{1}{4\sqrt{2}}g_Z(H+v)Z^\mu \left[\partial_\mu(\bar{t}t) + \bar{t}\gamma_5\partial_\mu t - (\partial_\mu \bar{t})\gamma_5 t - i\frac{4}{3}g_1 B_\mu \bar{t}\gamma_5 t \right] \\ & + \frac{1}{2}g_2(H+v)W_\mu^- \left[\bar{b}_L\partial^\mu t_R - i\frac{2}{3}g_1 B^\mu \bar{b}_L t_R \right] \end{aligned}$$

$$+\frac{1}{2}g_2(H+v)W_\mu^+ \left[(\partial^\mu \bar{t}_R)b_L + i\frac{2}{3}g_1 B^\mu \bar{t}_R b_L \right] \quad (\text{A.4})$$

$$\begin{aligned} O_{tW\Phi} = & i\frac{1}{2\sqrt{2}}(H+v)(\bar{t}\sigma^{\mu\nu}\gamma_5 t) \left[W_{\mu\nu}^3 - ig_2(W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) \right] \\ & + i\frac{1}{2}(H+v)(\bar{b}_L\sigma^{\mu\nu}t_R) \left[W_{\mu\nu}^- - ig_2(W_\mu^- W_\nu^3 - W_\mu^3 W_\nu^-) \right] \\ & - i\frac{1}{2}(H+v)(\bar{t}_R\sigma^{\mu\nu}b_L) \left[W_{\mu\nu}^+ - ig_2(W_\mu^3 W_\nu^+ - W_\mu^+ W_\nu^3) \right] \end{aligned} \quad (\text{A.5})$$

$$O_{tB\Phi} = i\frac{1}{\sqrt{2}}(H+v)(\bar{t}\sigma^{\mu\nu}\gamma_5 t)B_{\mu\nu} \quad (\text{A.6})$$

$$O_{tG\Phi} = i\frac{1}{\sqrt{2}}(H+v)(\bar{t}\sigma^{\mu\nu}\gamma_5 T^A t)G_{\mu\nu}^A \quad (\text{A.7})$$

$$\begin{aligned} O_{tG} = & i \left[\bar{t}_R \gamma^\mu T^A \partial^\nu t_R - \partial^\nu \bar{t}_R \gamma^\mu T^A t_R \right] G_{\mu\nu}^A \\ & + g_s \bar{t}_R \gamma^\mu \{G^\nu, G_{\mu\nu}\} t_R + \frac{4g_1}{3} \bar{t}_R \gamma^\mu G_{\mu\nu} B^\nu t_R \end{aligned} \quad (\text{A.8})$$

$$O_{tB} = i \left[\bar{t}_R \gamma^\mu \partial^\nu t_R - \partial^\nu \bar{t}_R \gamma^\mu t_R \right] B_{\mu\nu} + 2g_s \bar{t}_R \gamma^\mu G^\nu t_R B_{\mu\nu} + \frac{4}{3}g_1 \bar{t}_R \gamma^\mu t_R B_{\mu\nu} B^\nu \quad (\text{A.9})$$

(2) Class 2

$$\begin{aligned} O_{qG} = & i \left[\bar{q}_L \gamma^\mu T^A \partial^\nu q_L - \partial^\nu \bar{q}_L \gamma^\mu T^A q_L \right] G_{\mu\nu}^A \\ & + g_s \bar{q}_L \gamma^\mu \{G^\nu, G_{\mu\nu}\} q_L + 2g_2 \bar{q}_L \gamma^\mu W^\nu G_{\mu\nu} q_L + \frac{1}{3}g_1 \bar{q}_L \gamma^\mu G_{\mu\nu} B^\nu q_L \end{aligned} \quad (\text{A.10})$$

$$\begin{aligned} O_{qW} = & \frac{i}{2}W_{\mu\nu}^3 \left[\bar{t}_L \gamma^\mu \partial^\nu t_L - \partial^\nu \bar{t}_L \gamma^\mu t_L - \bar{b}_L \gamma^\mu \partial^\nu b_L + \partial^\nu \bar{b}_L \gamma^\mu b_L \right] \\ & + \frac{i}{\sqrt{2}} \left[W_{\mu\nu}^+ (\bar{t}_L \gamma^\mu \partial^\nu b_L - \partial^\nu \bar{t}_L \gamma^\mu b_L) + W_{\mu\nu}^- (\bar{b}_L \gamma^\mu \partial^\nu t_L - \partial^\nu \bar{b}_L \gamma^\mu t_L) \right] \\ & + g_2 \bar{q}_L \gamma^\mu [W_\mu, W_\nu] \partial^\nu q_L - g_2 \partial^\nu \bar{q}_L \gamma^\mu [W_\mu, W_\nu] q_L \\ & + 2g_s \bar{q}_L \gamma^\mu G^\nu W_{\mu\nu} q_L + \frac{1}{2}g_2 (\vec{W}_{\mu\nu} \cdot \vec{W}^\nu) \bar{q}_L \gamma^\mu q_L + \frac{1}{3}g_1 B^\nu \bar{q}_L \gamma^\mu W_{\mu\nu} q_L \end{aligned} \quad (\text{A.11})$$

$$O_{qB} = iB_{\mu\nu} \left[\bar{q}_L \gamma^\mu \partial^\nu q_L - \partial^\nu \bar{q}_L \gamma^\mu q_L - 2i\bar{q}_L \gamma^\mu (g_s G^\nu + g_2 W^\nu + \frac{1}{6}g_1 B^\nu) q_L \right] \quad (\text{A.12})$$

$$\begin{aligned} O_{bG} = & i \left[\bar{b}_R \gamma^\mu T^A \partial^\nu b_R - \partial^\nu \bar{b}_R \gamma^\mu T^A b_R \right] G_{\mu\nu}^A \\ & + g_s \bar{b}_R \gamma^\mu \{G^\nu, G_{\mu\nu}\} b_R - \frac{2g_1}{3} \bar{b}_R \gamma^\mu G_{\mu\nu} B^\nu b_R \end{aligned} \quad (\text{A.13})$$

$$O_{bB} = i \left[\bar{b}_R \gamma^\mu \partial^\nu b_R - \partial^\nu \bar{b}_R \gamma^\mu b_R \right] B_{\mu\nu} + 2g_s \bar{b}_R \gamma^\mu G^\nu b_R B_{\mu\nu} - \frac{2}{3}g_1 \bar{b}_R \gamma^\mu b_R B_{\mu\nu} B^\nu \quad (\text{A.14})$$

$$O_{\Phi q}^{(1)} = (H+v)\partial_\mu H \left[\bar{t}_L \gamma^\mu t_L + \bar{b}_L \gamma^\mu b_L \right] \quad (\text{A.15})$$

$$O_{\Phi q}^{(3)} = -O_{\Phi q}^{(1)} + 2(H+v)\partial_\mu H \bar{b}_L \gamma^\mu b_L$$

$$-\frac{i}{\sqrt{2}}g_2(H+v)^2(W_\mu^+\bar{t}_L\gamma^\mu b_L - W_\mu^-\bar{b}_L\gamma^\mu t_L) \quad (\text{A.16})$$

$$O_{\Phi b} = (H+v)\partial_\mu H\bar{b}_R\gamma^\mu b_R \quad (\text{A.17})$$

$$O_{b1} = \frac{1}{2\sqrt{2}}H(H+v)(H+2v)\bar{b}i\gamma_5 b \quad (\text{A.18})$$

$$\begin{aligned} O_{Db} = & i\frac{1}{2\sqrt{2}}\partial^\mu H \left[\bar{b}\partial_\mu b - (\partial_\mu \bar{b})b + \partial_\mu(\bar{b}\gamma_5 b) + i\frac{2}{3}g_1 B_\mu(\bar{b}b) \right] \\ & + \frac{1}{4}g_Z(H+v)Z^\mu \left[\partial_\mu(\bar{b}b) + \bar{b}\gamma_5\partial_\mu b - (\partial_\mu \bar{b})\gamma_5 b + \frac{2}{3}g_1 B_\mu(\bar{b}i\gamma_5 b) \right] \\ & + \frac{g_2}{2}(H+v) \left[W_\mu^+(\bar{t}_L\partial_\mu b_R + \frac{i}{3}g_1 B_\mu\bar{t}_L b_R) + W_\mu^-(\partial_\mu \bar{b}_R t_L - \frac{i}{3}g_1 B_\mu\bar{b}_R t_L) \right] \end{aligned} \quad (\text{A.19})$$

$$\begin{aligned} O_{bW\Phi} = & i\frac{1}{2}(H+v) \left[W_{\mu\nu}^+(\bar{t}_L\sigma^{\mu\nu} b_R) - W_{\mu\nu}^-(\bar{b}_R\sigma^{\mu\nu} t_L) - \frac{1}{\sqrt{2}}W_{\mu\nu}^3(\bar{b}\sigma^{\mu\nu}\gamma_5 b) \right. \\ & + ig_2(W_\mu^+W_\nu^3 - W_\mu^3W_\nu^+)(\bar{t}_L\sigma^{\mu\nu} b_R) + ig_2(W_\mu^-W_\nu^3 - W_\mu^3W_\nu^-)(\bar{b}_R\sigma^{\mu\nu} t_L) \\ & \left. + i\frac{g_2}{\sqrt{2}}(W_\mu^+W_\nu^- - W_\mu^-W_\nu^+)(\bar{b}\sigma^{\mu\nu}\gamma_5 b) \right] \end{aligned} \quad (\text{A.20})$$

$$O_{bB\Phi} = \frac{i}{\sqrt{2}}(H+v)B_{\mu\nu}(\bar{b}\sigma^{\mu\nu}\gamma_5 b) \quad (\text{A.21})$$

$$O_{bG\Phi} = \frac{i}{\sqrt{2}}(H+v)G_{\mu\nu}^A(\bar{b}\sigma^{\mu\nu}\gamma_5 T^A b) \quad (\text{A.22})$$

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Figure Captions

Fig. 1 The asymmetry between the degrees of transverse polarization of the top quark and top antiquark induced by O_{qW} as a function of θ_t in top pair production at the NLC for $\sqrt{s} = 500$ GeV.

Fig. 2 The asymmetry between the degrees of transverse polarization of the top quark and top antiquark induced by O_{qW} as a function of θ_t in top pair production at the NLC for $\sqrt{s} = 1$ TeV.

Table 1

The contribution status of dimension-six CP-violating operators to electroweak and $gt\bar{t}$ couplings. The contribution of a CP-violating operator to a particular vertex is marked by \times .

	$Wt\bar{b}$	$Zt\bar{t}$	$\gamma t\bar{t}$	$Ht\bar{t}$	$gt\bar{t}$	$Zb\bar{b}$	$\gamma b\bar{b}$	$Hb\bar{b}$
O_{t1}				\times				
O_{t2}				\times				
O_{t3}	\times							
O_{Dt}	\times	\times		\times				
$O_{tW\Phi}$	\times	\times	\times					
$O_{tB\Phi}$		\times	\times					
$O_{tG\Phi}$					\times			
O_{tG}					\times			
O_{tB}		\times	\times					
O_{qG}					\times			
O_{qW}	\times	\times	\times			\times	\times	
O_{qB}		\times	\times			\times	\times	
O_{bB}						\times	\times	
$O_{\Phi q}^{(1)}$				\times				\times
$O_{\Phi q}^{(3)}$	\times			\times				\times
$O_{\Phi b}$								\times
O_{b1}								\times
O_{Db}	\times					\times		\times
$O_{bW\Phi}$	\times					\times	\times	
$O_{bB\Phi}$						\times	\times	

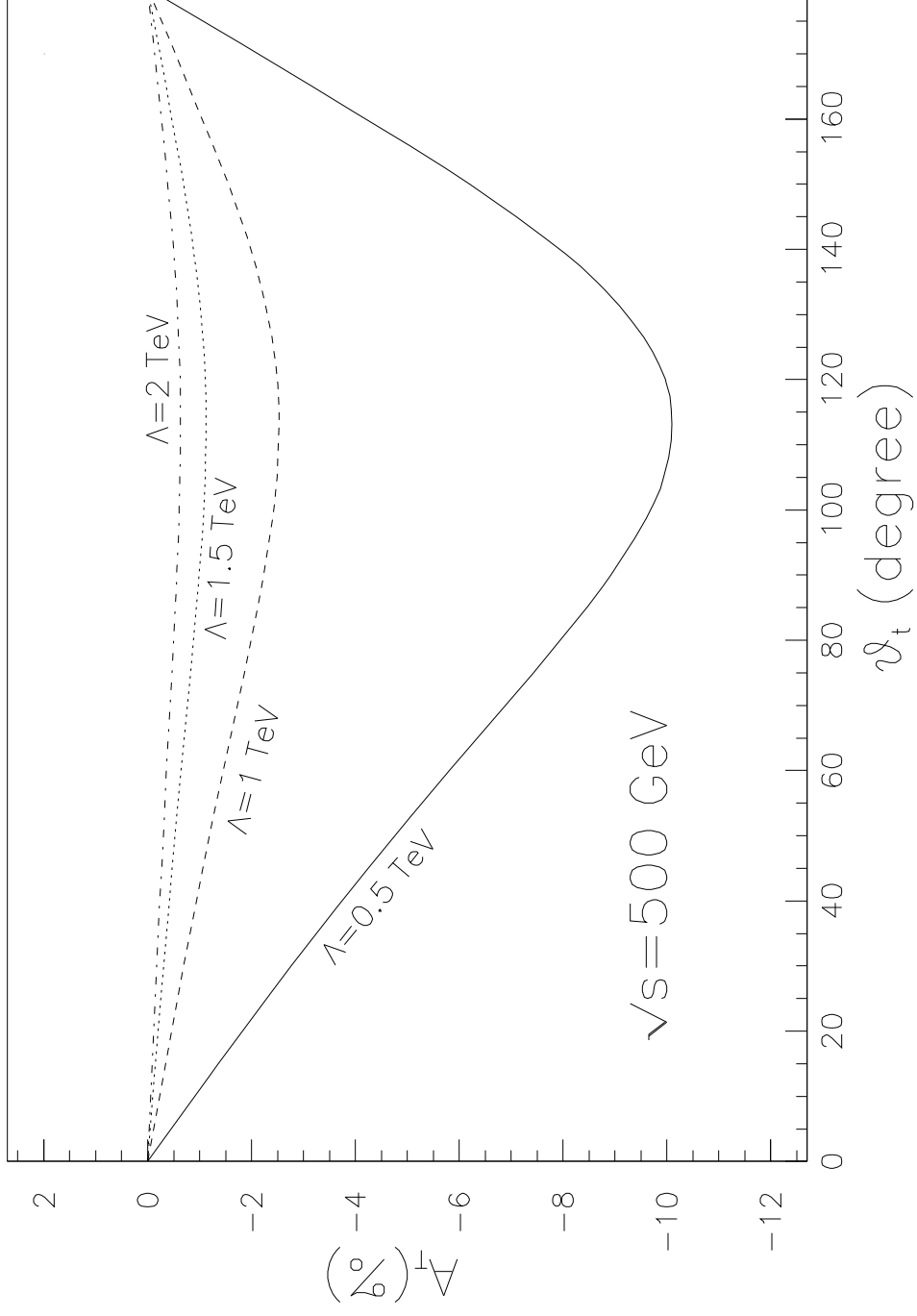


Fig.1

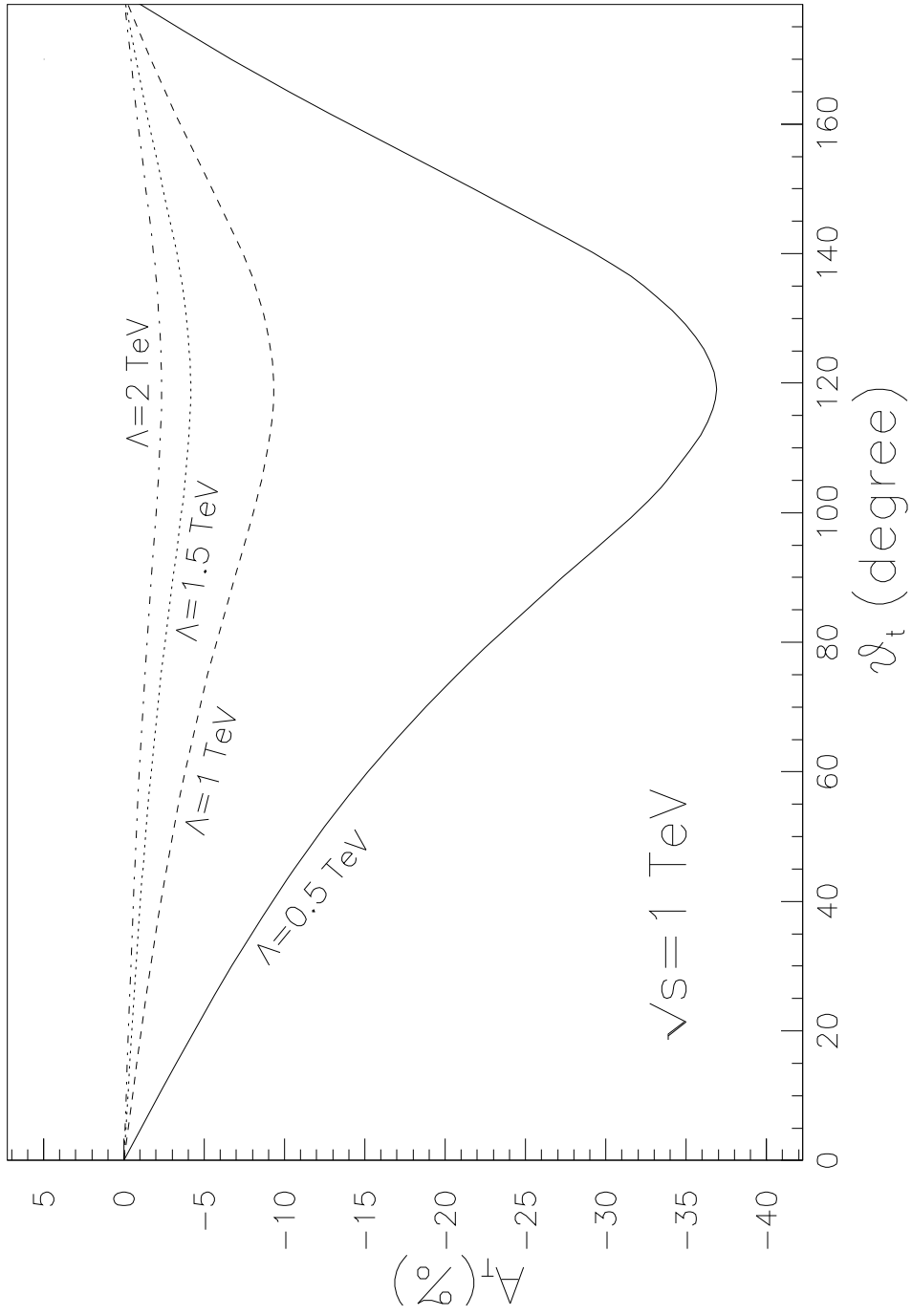


Fig.2